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Supersymmetric Effects on Isospin Symmetry Breaking and Direct CP Violation in $B \to \rho\gamma$

A. Ali, L.T. Handoko
Deutsches Elektronen Synchrotron DESY, Hamburg
and
D. London
Laboratoire René J.-A. Lévesque, Université de Montréal,
C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7

Abstract

We argue that one can search for physics beyond the standard model through measurements of the isospin-violating quantity $\Delta^{-0} \equiv \Gamma(B^- \to \rho^-\gamma)/2\Gamma(B^0 \to \rho^0\gamma) - 1$, its charge conjugate Δ^{+0}, and direct CP violation in the partial decay rates of $B^\pm \to \rho^\pm\gamma$. We illustrate this by working out theoretical profiles of the charge-conjugate averaged ratio $\Delta \equiv \frac{1}{2}(\Delta^{+0} + \Delta^{-0})$ and the CP asymmetry $\mathcal{A}_{CP}(B^\pm \to \rho^\pm\gamma)$ in the standard model and in some variants of the minimal supersymmetric standard model. We find that chargino contributions in the large $\tan\beta$ region may modify the magnitudes and flip the signs of Δ and $\mathcal{A}_{CP}(B^\pm \to \rho^\pm\gamma)$ compared to their standard-model values, providing an unmistakeable signature of supersymmetry.

Measurements of the radiative decays $B \to K^*\gamma$ [1] and $B \to X_s\gamma$ [2] have triggered a large number of theoretical studies whose aim is to provide precision tests of the flavor sector in the standard model (SM), and to search for possible hints of new physics, particularly supersymmetry [3]. The related Cabibbo-suppressed decays $B \to \rho\gamma$, $B \to \omega\gamma$ and $B \to X_d\gamma$, for which experiments have so far provided only upper bounds [4], but which surely will be measured at B-factories, have also been studied at great length. Within the SM, these latter decays are particularly interesting because they potentially allow us to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{td}, or, more generally, the quark mixing parameters $\bar{\rho}$ and $\bar{\eta}$ of the Wolfenstein parametrization of the CKM matrix [5]. While the inclusive decay is theoretically more robust [6], it is experimentally very challenging. In view of this, considerable effort has gone into consolidating the theoretical profile of the exclusive decays $B \to V\gamma$ ($V = K^*, \rho, \omega$) in the SM [7, 8, 9, 10, 11, 12, 13].

In this letter, we argue that the interference of the short-distance (SD) penguin amplitude and long-distance (LD) tree amplitude in exclusive radiative B-decays, which is often considered as an *impediment* to a precise determination of the CKM parameters from their branching ratios, may turn out to be *a boon in disguise* in searching for new physics. To illustrate this point, we focus on the decays $B^0(\overline{B^0}) \to \rho^0\gamma$ and $B^\pm \to \rho^\pm\gamma$, and consider the isospin-violating ratio defined as

$$\Delta^{-0} \equiv \frac{\Gamma(B^- \to \rho^-\gamma)}{2\Gamma(B^0 \to \rho^0\gamma)} - 1 \, , \tag{1}$$

along with its charge conjugate Δ^{+0}. (Since theoretical estimates give $\tau(B^\pm) = \tau(B^0)$, to within a couple of percent, and the present data support this conclusion [14], the quantities $\Delta^{\pm 0}$ can be interpreted in terms of the branching ratios.) Note that the ratios $\Delta^{\pm 0}$ deviate from zero (their isospin limit) due to the SD-LD interference effects mentioned above.

We compare the profiles of the charge-conjugate averaged ratio $\Delta \equiv \frac{1}{2}(\Delta^{+0} + \Delta^{-0})$ in the SM and in a class of variants of the minimal supersymmetric standard model (MSSM) in which all non-diagonal flavor transitions take place essentially via the CKM quark mixing matrix. Although the SM and the MSSM yield similar values of Δ in some regions of parameter space, in other regions the MSSM may change this ratio significantly. This can happen in two different ways. First, in some MSSM models a larger value of the angle α in the unitarity triangle is preferred [15]. Since Δ increases with α in the quadrant $\pi/2 \le \alpha \le \pi$ [9, 10], the ratio Δ may be enhanced in the MSSM. The second effect, which is particularly striking, is that the sign of Δ can be flipped in MSSM models. This can happen in that region of large $\tan\beta$ supersymmetric parameter space in which the chargino-stop contributions are known to flip the sign of the effective matrix elements of the electromagnetic and chromomagnetic penguin operators [16, 17, 18, 19].

Finally, we also consider the direct CP asymmetry $\mathcal{A}_{\rm CP}$ in the decays $B^\pm \to \rho^\pm\gamma$. (The direct CP asymmetries in $B^0(\overline{B^0}) \to \rho^0\gamma$ and $B^0(\overline{B^0}) \to \omega\gamma$ are very similar to $\mathcal{A}_{\rm CP}(B^\pm \to \rho^\pm\gamma)$, though their time evolution will be modulated by B^0–$\overline{B^0}$ mixing effects.) We find that, for MSSM's with large $\tan\beta$, the sign of $\mathcal{A}_{\rm CP}$ may turn out to be opposite that of the SM.

Having summarized our main results above, we now present the calculation. To compute the radiative weak transitions ($b \to d\gamma$), we use the effective Hamiltonian

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left[\lambda_u^{(d)} (\mathcal{C}_1(\mu)\mathcal{O}_1(\mu) + \mathcal{C}_2(\mu)\mathcal{O}_2(\mu)) - \lambda_d^{(t)} \mathcal{C}_7^{eff}(\mu)\mathcal{O}_7(\mu) + ... \right] . \tag{2}$$

Here, $\lambda_q^{(q')} = V_{qb}V_{qq'}^*$ are the CKM factors, and we have restricted ourselves to those contributions which will be important in what follows. The operators $\mathcal{O}_1(\mu)$ and $\mathcal{O}_2(\mu)$ are the four-quark operators

$$\mathcal{O}_1 = (\bar{d}_\alpha \Gamma^\mu u_\beta)(\bar{u}_\beta \Gamma_\mu b_\alpha) \quad , \qquad \mathcal{O}_2 = (\bar{d}_\alpha \Gamma^\mu u_\alpha)(\bar{u}_\beta \Gamma_\mu b_\beta) \ , \tag{3}$$

where $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$, α and β are the SU(3) color indices, and $\mathcal{C}_1$ and $\mathcal{C}_2$ are the corresponding Wilson coefficients. $\mathcal{O}_7$ is the magnetic moment operator

$$\mathcal{O}_7 = \frac{em_b}{8\pi^2} \bar{d}\sigma^{\mu\nu}(1-\gamma_5)F_{\mu\nu}b \ , \tag{4}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor. We note that the coefficient $\mathcal{C}_7^{eff}(\mu)$ also includes the effect of the four-quark operators $\mathcal{O}_5$ and $\mathcal{O}_6$, and the operator matrix elements and their coefficients are calculated at the b-quark mass scale $\mu = m_b$. For further details and definitions, see Ref. [9].

The decay amplitudes of interest can be written in the form:

$$\begin{aligned} \mathcal{M}(B^- \to \rho^- \gamma) &= \lambda_t^{(d)} a_P \Big(1 - \frac{|\lambda_u^{(d)}|}{|\lambda_t^{(d)}|} R_L^{(-)} e^{i\alpha}\Big) \ , \\ \mathcal{M}(B^0 \to \rho^0 \gamma) &= \lambda_t^{(d)} a_P \Big(1 - \frac{|\lambda_u^{(d)}|}{|\lambda_t^{(d)}|} R_L^{(0)} e^{i\alpha}\Big) \ , \end{aligned} \tag{5}$$

where isospin symmetry has been used in writing $a_P^{(-)} = a_P^{(0)} \equiv a_P$ for the penguin amplitudes, and α is one of the angles of the CKM unitarity triangle. The dynamical quantities $R_L^{(-)}$ and $R_L^{(0)}$, which are in general complex due to strong interactions, are the ratios of the reduced LD and SD amplitudes in the decays $B^- \to \rho^-\gamma$ and $B^0 \to \rho^0\gamma$, respectively. In general, a_P, $R_L^{(-)}$ and $R_L^{(0)}$ are all model-dependent.

Light-cone QCD sum rules, which take into account the dominant $W^\pm$-annihilation and $W^\pm$-exchange contributions, typically yield $R_L^{(-)} \simeq -0.3 \pm 0.07$ and $R_L^{(0)} \simeq 0.03 \pm 0.01$ [8, 9]. These estimates, which are obtained using the factorization approximation, have been essentially confirmed by a recent calculation in which non-factorizable corrections are proven to vanish in the chiral limit to leading twist, in the heavy quark limit [13]. In addition, long-distance contributions from other topologies have been estimated systematically and found to be small [11, 12, 13]. Eventually, radiative decays $B^\pm \ell^\pm \nu\gamma$ can be used to compute the leading ($W^\pm$-exchange) topologies in a model-independent way [13]. Of course, one still needs to know a_P to get the branching ratios.

The expression for the ratio of the branching ratios of interest can be written as

$$\frac{\mathcal{B}(B^- \to \rho^-\gamma)}{\mathcal{B}(B^0 \to \rho^0\gamma)} \simeq \left| 1 + \frac{4\pi^2 m_{\rho^\pm}}{m_b} \frac{\mathcal{C}_2 + \mathcal{C}_1/N_c}{\mathcal{C}_7^{eff}} r_u^{\rho^\pm} \frac{\lambda_u^{(d)}}{\lambda_t^{(d)}} \right|^2 \ , \tag{6}$$

where $r_u^{\rho^\pm}$ lumps together the dominant (W-annihilation) and possible sub-dominant LD contributions. Borrowing the notation from Ref. [13],

$$\epsilon_A \mathrm{e}^{i\phi_A} \equiv \frac{4\pi^2 m_{\rho^\pm}}{m_b} \frac{\mathcal{C}_2 + \mathcal{C}_1/N_c}{\mathcal{C}_7^{eff}} r_u^{\rho^\pm} \ , \tag{7}$$

and noting that $\lambda_u^{(d)}/\lambda_t^{(d)} = -|\lambda_u^{(d)}/\lambda_t^{(d)}|e^{+i\alpha}$, which holds in the SM and in the MSSM models being considered here, the isospin breaking ratios [Eq. (1)] can be expressed as

$$\Delta^{\pm 0} = 2\epsilon_A \left[\cos\phi_A F_1 \mp \sin\phi_A F_2 + \frac{1}{2}\epsilon_A (F_1^2 + F_2^2)\right] \ . \tag{8}$$

Here, $F_{1,2}$ are (implicit) functions of the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$:

$$F_1 = -\left|\frac{\lambda_u^{(d)}}{\lambda_t^{(d)}}\right| \cos\alpha \ , \quad F_2 = -\left|\frac{\lambda_u^{(d)}}{\lambda_t^{(d)}}\right| \sin\alpha \ , \tag{9}$$

with $(F_1^2 + F_2^2) = \left|\lambda_u^{(d)}/\lambda_t^{(d)}\right|^2$.

The charge-conjugated averaged ratio, defined as $\Delta \equiv \frac{1}{2}[\Delta^{-0} + \Delta^{+0}]$, has the following leading-order (LO) expression:

$$\Delta_{\rm LO} = 2\epsilon_A \left[\cos\phi_A F_1 + \frac{1}{2}\epsilon_A (F_1^2 + F_2^2)\right] \simeq 2\epsilon_A \left[F_1 + \frac{1}{2}\epsilon_A (F_1^2 + F_2^2)\right] \ , \tag{10}$$

where the near equality reflects that, in this approximation, the strong interaction phase ϕ_A disappears in the chiral limit [13].

In fact, one can go to next-to-leading-order (NLO) in the calculation of the above quantities. The NLO-corrected expression for the branching ratios and Δ can be derived from the corresponding calculations for the inclusive decay $B \to X_s\gamma$ [20, 21] and $B \to X_d\gamma$ [6]:

$$\begin{aligned}
\Gamma(B^\pm \to \rho^\pm\gamma) \ &= \ \frac{G_F^2 \alpha |\lambda_t^{(d)}|^2}{32\pi^4} m_B^5 \left(1 - \frac{m_\rho^2}{m_B^2}\right)^3 |T_1^\rho|^2 \left\{\left|\mathcal{C}_7^{(0)eff} + A_R^{(1)t}\right|^2\right. \\
&+ \left(F_1^2 + F_2^2\right)\left(|A_R^u + L_R^u|^2\right) \\
&+ 2F_1 \left[\mathcal{C}_7^{(0)eff}\left(A_R^u + L_R^u\right) + A_R^{(1)t} L_R^u\right] \\
&\left.\mp 2F_2 \left[\mathcal{C}_7^{(0)eff} A_I^u - A_I^{(1)t} L_R^u\right]\right\} \ .
\end{aligned} \tag{11}$$

Here G_F is the Fermi coupling constant, $\alpha = \alpha(0) = 1/137$, T_1^ρ is the $B \to \rho$ form factor involving the magnetic moment operator $\mathcal{O}_7$, evaluated at $q^2 = 0$, and $L_R^u = \epsilon_A \mathcal{C}_7^{(0)eff}$. The quantities $A_{R,I}^{(1)t}$ and $A_{R,I}^u$ represent the real and imaginary parts of the explicit $O(\alpha_s)$ contributions to the matrix elements evaluated at a scale μ:

$$\begin{aligned}
A^{(1)t} \ &= \ \frac{\alpha_s(\mu)}{4\pi}\left\{C_7^{(1)}(\mu) - \frac{16}{3} C_7^{(0)eff}(\mu)\right. \\
&\left. + \sum_i^8 C_i^{(0)eff}(\mu) \left[\gamma_{i7}^{(0)} \ \ln\frac{m_b}{\mu} + r_i(z)\right]\right\} \ ,
\end{aligned} \tag{12}$$

$$A^u \ = \ \frac{\alpha_s(\mu)}{4\pi} C_2^{(0)}(\mu) \ [r_2(z) - r_2(0)] \ , \tag{13}$$

where r_i's are complex numbers. Expressions for the various quantities appearing in the above equations can be found in Refs. [20, 21]. We stress that the gluon bremsstrahlung parts have been dropped in calculating $\Gamma(B \to \rho\gamma)$, except those needed to cancel the divergence in the $O(\alpha_s)$ virtual corrections in the decay $b \to d\gamma$. Note that, in the above rate, all terms higher than $O(\alpha_s)$ have to be dropped for theoretical consistency. The expression for $\Gamma(B^0 \to \rho^0\gamma)$ can be obtained by obvious replacements, except that $L_R^u(B^0) \ll L_R^u(B^\pm)$.

Using the above expression, the NLO isospin-violating ratio Δ is found to be:

$$\begin{aligned}\Delta_{\rm NLO} &= \Delta_{\rm LO} \\ &-\frac{2\epsilon_A}{\mathcal{C}_7^{(0)eff}}\left[F_1 A_R^{(1)t} - \left(F_2^2 - F_1^2\right) A_R^u + \epsilon_A \left(F_1^2 + F_2^2\right)\left(A_R^{(1)t} + F_1 A_R^u\right)\right] \quad (14)\end{aligned}$$

where $\Delta_{\rm LO}$ is given in Eq. (10).

The values for the various input quantities used in the numerical calculations of $\Delta_{\rm LO}$ and $\Delta_{\rm NLO}$ in the SM are as follows: $\mathcal{C}_7^{(0)eff}(m_b) = -0.318$, $A_R^{(1)t} = -0.022$, $A_R^u = +0.049$, and $\epsilon_A = -0.3$. The remaining ingredient is a determination of the allowed ranges for the functions F_1 and F_2. Taking into account the present experimental and theoretical constraints on the parameters of the CKM matrix, the profile of the unitarity triangle in the SM was presented by two of us in Ref. [15]. In Fig. 1 we show the allowed F_1–α and F_2–α correlations at 95% C.L. In these figures, the SM plots are found on the left-hand side, and are labelled by $f = 0.0$. The ranges of the hadronic parameters $f_{B_d}\sqrt{\hat{B}_{B_d}}$ and B_K used in these fits are indicated on top of the figures. (For definitions and further discussions, see Ref. [15].) Note that the CP phase α is constrained to lie in the range $75^\circ \le \alpha \le 121^\circ$ at 95% C.L. [15].

With this information, we can now calculate the ratios $\Delta_{\rm LO}$ and $\Delta_{\rm NLO}$ in the SM. In Fig. 2 the results are shown for these quantities as a function of the angle α. In these figures we have assumed that $|V_{ub}/V_{td}| = 0.48$ (its central value [15]). However, for a given value of α, $\Delta_{\rm LO}$ and $\Delta_{\rm NLO}$ may in fact take a range of values. This residual CKM-related range is given essentially by the F_1–α correlation presented in the upper-left plot in Fig. 1. Note that the isospin-violating ratio is very stable against NLO corrections in the SM. This observation, together with the discussions earlier about the determination of ϵ_A, makes Δ suitable for precision tests of the SM. In particular, its measurement will determine α in the SM.

We now turn to the direct CP asymmetry [22]. As noted earlier, the strong interaction phase ϕ_A of Eq. (7) disappears in the chiral limit [13], which implies that, to lowest order, there is no CP-violation in the decay rates for $B \to \rho\gamma$. Therefore, the strong phases in the exclusive decays $B^\pm \to \rho^\pm\gamma$ and $B^0(\overline{B^0}) \to \rho^0\gamma$, which are necessary for inducing direct CP-violation, must be generated by higher-order perturbative QCD corrections. Concentrating on the charged B decays, we define the CP asymmetry as

$$\mathcal{A}_{\rm CP} \equiv \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)} \ . \quad (15)$$

Since, in the heavy quark limit, there are no non-factorizing strong phases in the W-annihilation part of the $B \to \rho\gamma$ amplitudes [13], the strong phases are generated *entirely* by the Bander-Silverman-Soni mechanism [23], which involves the interference of the penguin operator $\mathcal{O}_7$ and the four-quark operator $\mathcal{O}_2$ [24, 25]. This mechanism has been employed by Greub, Simma and Wyler to calculate $\mathcal{A}_{\rm CP}$, using a wave function model [25] for the mesons. Since we are working to leading twist, we shall ignore the effects involving virtual corrections off the spectator quarks, arguing that they are suppressed by powers of $1/m_b$. In that case, the CP asymmetry is determined by perturbation theory up to a non-perturbative quantity which can be determined from the ratio Δ. The expression for $\mathcal{A}_{\rm CP}$ is given by

$$\mathcal{A}_{\rm CP} \;=\; -\frac{2F_2}{\mathcal{C}_7^{(0)eff}(1+\Delta_{\rm LO})}\left[A_I^u - \epsilon_A A_I^{(1)t}\right] \; . \tag{16}$$

The quantities $A_I^{(1)t}$ and A_I^u take the values $A_I^{(1)t} = -0.016$ in the SM and $A_I^u = +0.046$ in both the SM and MSSM.

Note that Δ and $\mathcal{A}_{\rm CP}$ are complementary measurements. Dropping the small $O(\epsilon_A^2)$ terms in Eq. (14), we see that Δ is essentially proportional to F_1, while $\mathcal{A}_{\rm CP}$ is proportional to F_2. Thus, for $\alpha \simeq \pi/2$, Δ is very small, while $\mathcal{A}_{\rm CP}$ takes its maximal value. Conversely, if the value of α is far from $\pi/2$, the CP asymmetry decreases, while Δ becomes measurable.

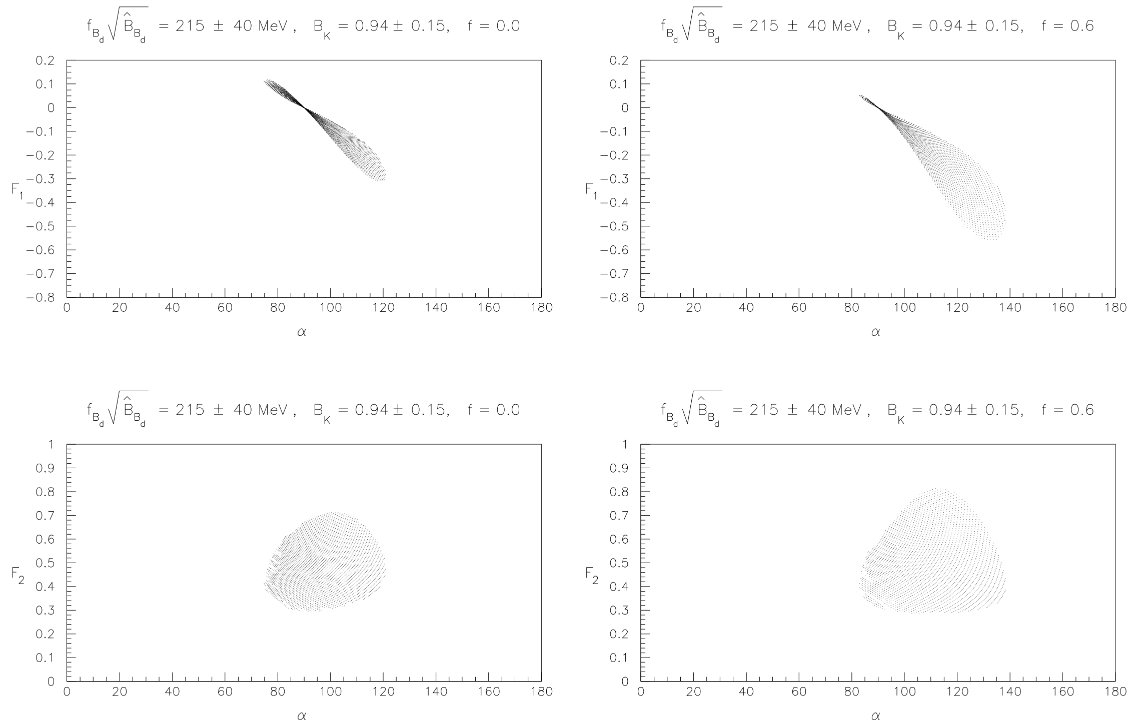


Figure 1: Upper-left: SM F_1–α correlation. Upper-right: MSSM F_1–α correlation. Lower-left: SM F_2–α correlation. Lower-right: MSSM F_2–α correlation. The parameters used in calculating the correlations are indicated on top of the figures.

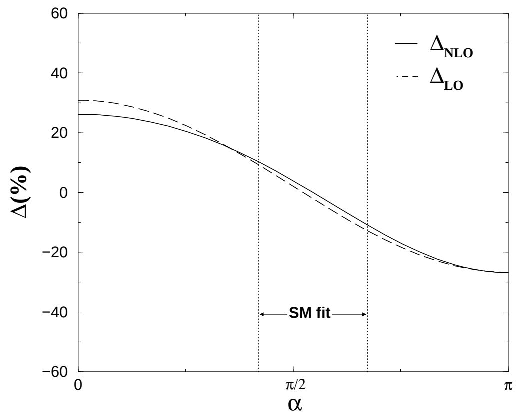


Figure 2: The isospin-violating ratio Δ at LO and NLO in the SM. We have set $\epsilon_A = -0.3$. The curves correspond to the central value of the CKM fits $|V_{ub}/V_{td}| = 0.48$ [15].

We are now ready to examine the supersymmetric contributions to Δ and $\mathcal{A}_{\rm CP}$. To begin with, we note that the NLO corrections to the decays $B \to X_s\gamma$ have been calculated in only one particular realization of the MSSM, the so-called *minimal flavor violation* scenario [17]. While this calculation considers an important parameter space in the MSSM, it nevertheless neglects other contributions, such as those from gluinos, which are important in other regions of parameter space [18]. In the small-$\tan\beta$ domain, where the neglected contributions are small, we have numerically calculated the NLO quantities and found that the NLO correction to Δ in the MSSM with minimal flavor violation is very similar to that in the SM, and hence unimportant. The complete NLO corrections for the large-$\tan\beta$ case, including gluino contributions, are not yet available. Hence, in comparing the SM profile with that of the MSSM, we shall restrict ourselves to $\Delta_{\rm LO}$.

Supersymmetry can affect Δ and $\mathcal{A}_{\rm CP}$ in two distinct ways. First, the allowed values of the functions F_1 and F_2 are different in the MSSM. We recall that the supersymmetric contributions to the mass differences $M_{12}(B)$ and $M_{12}(K)$ can be written as follows (for details and references, see Ref. [15]):

$$\begin{aligned}
\Delta M_d &= \Delta M_d(SM)[1 + f_d(m_{\chi_2^\pm}, m_{\tilde{t}_R}, m_{H^\pm}, \tan\beta)], \\
\Delta M_s &= \Delta M_s(SM)[1 + f_s(m_{\chi_2^\pm}, m_{\tilde{t}_R}, m_{H^\pm}, \tan\beta)], \\
|\epsilon| &= \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} \hat{B}_K \left(A^2\lambda^6\bar{\eta}\right) \left(y_c \left\{\hat{\eta}_{ct} f_3(y_c, y_t) - \hat{\eta}_{cc}\right\}\right. \\
&\quad \left. + \, \hat{\eta}_{tt} y_t f_2(y_t)[1 + f_\epsilon(m_{\chi_2^\pm}, m_{\tilde{t}_2}, m_{H^\pm}, \tan\beta)] A^2\lambda^4(1-\bar{\rho})\right). \qquad (17)
\end{aligned}$$

To an excellent approximation, one has $f_d = f_s = f_\epsilon \equiv f$. The quantity f is a function of the masses of the (lighter) right-handed top squark ($m_{\tilde{t}_R}$), chargino ($m_{\tilde{\chi}_2^\pm}$) and the charged Higgs ($m_{H^\pm}$), as well as of $\tan\beta$. The maximum allowed value of f depends on the model. Typical values are: minimal supergravity ($f = 0.2$), non-minimal supergravity ($f = 0.4$) [19], and MSSM with constraints from electric dipole moments (EDM's) ($f = 0.6$) [26]. The plots in the upper right-hand and

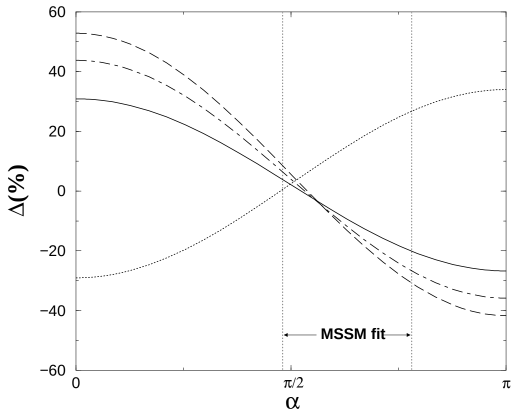
lower right-hand corners of Fig. 1 show the allowed F_1–α and F_2–α correlations, respectively, for the MSSM with $f = 0.6$. We see that these correlations can be measurably different from the SM. In particular, much larger values of α are allowed compared to the SM. Thus, for $f = 0.6$, the fits yield $86^\circ \leq \alpha \leq 141^\circ$ at 95% C.L.

The second way in which supersymmetry affects Δ and $\mathcal{A}_{\rm CP}$ is via the Wilson coefficients. In contrasting the SM and MSSM profiles, we assume, as per the usual expectations, that the coefficients of the tree amplitudes, $\mathcal{C}_1^{(0)}$ and $\mathcal{C}_2^{(0)}$, are the same in these models, but that $\mathcal{C}_7^{(0)eff}(\mu)$ may differ. This latter coefficient is constrained by the measured branching ratio of the decay $B \to X_s\gamma$, yielding a bound $2.0 \times 10^{-4} \leq \mathcal{B}(B \to X_s\gamma) \leq 4.5 \times 10^{-4}$ at 95% C.L. [2]. The resulting constraints on the magnitude and phase of the ratio $\mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}$ in the context of the MSSM being considered can be summarized as follows. In the absence of the constraints on the EDM's of the neutron and electron, the real and imaginary parts of $\mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}$ can vary substantially. But if one takes into account the EDM constraints, the imaginary part of $\mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}$ is highly suppressed. However, depending on the value of $\tan\beta$, both positive- and negative-valued solutions of $\mathcal{C}_7$ are allowed. For example, a recent analysis of $\mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}$ in the minimal supergravity model yields values in the ranges $0.7 \leq {\rm Re}[\mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}] \leq 1.2$ for small $\tan\beta$ (say $\tan\beta \leq 10$), but for larger values of $\tan\beta$, negative values of this ratio are admissible. Thus, for $\tan\beta = 30$, a range $-1.5 \leq \mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})} \leq -0.8$ is allowed by present data [19].

As for $\mathcal{A}_{\rm CP}(B^\pm \to \rho^\pm\gamma)$, the dominant contribution, proportional to A_I^u, is identical for the SM and MSSM. However, the value of the quantity $A_I^{(1)t}$ in the MSSM depends on the region of parameter space considered. For small $\tan\beta$, $A_I^{(1)t}$ has almost the same value as in the SM, while for large $\tan\beta$, it may appreciably differ from the SM value. However, since its contribution to $\mathcal{A}_{\rm CP}(B^\pm \to \rho^\pm\gamma)$ is suppressed due to the ϵ_A factor, its precise value is not so important numerically. In calculating $\mathcal{A}_{\rm CP}$ in the MSSM, we have set $A_I^{(1)t} = 0$.

In Fig. 3 we contrast the expectations for Δ in the SM and in two variants of the MSSM, characterized by small and large values of $\tan\beta$. The residual CKM-related range in the allowed values of Δ is again given essentially by the F_1–α correlation presented earlier in the upper two plots of Fig. 1 for the SM ($f = 0.0$) and MSSM ($f = 0.6$). Note that if the large $\tan\beta$ MSSM solution is realized in nature, the measured value of $\Delta^{\pm 0}$ can be markedly different than in the SM. More importantly, since $\mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}$ can be negative, the sign of $\Delta^{\pm 0}$ may change over a large region of the allowed CKM parameter space. This would be a striking signature of new physics, and would strongly suggest the presence of supersymmetry.

Similar effects appear in the CP asymmetry $\mathcal{A}_{\rm CP}$. Since, in the models being considered here, there are no other phases at this order, Eq. (16) holds in the MSSM, with the proviso that numerically $\mathcal{C}_7^{(0)eff}$ now depends on the parameters of the MSSM. In particular, for large $\tan\beta$, the CP asymmetry can be significantly larger than the one in the SM. Again, since $\mathcal{C}_7^{(0)eff}(MSSM) \simeq -\mathcal{C}_7^{(0)eff}(SM)$ is allowed, the CP asymmetry $\mathcal{A}_{\rm CP}(B^\pm \to \rho^\pm\gamma)$ reverses sign in this case (as does the asymmetry in the inclusive decays $\mathcal{A}_{\rm CP}(B^\pm \to X_d^\pm\gamma)$). This is illustrated in Fig. 3.



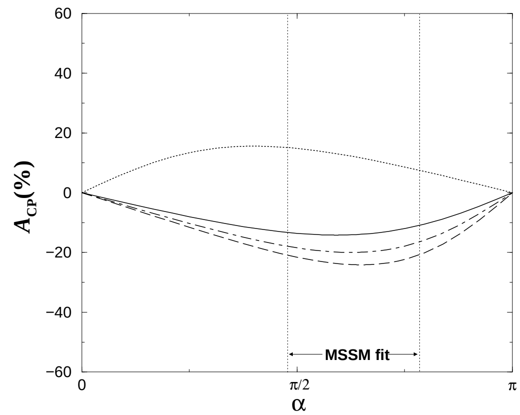


Figure 3: The isospin-violating ratio Δ in LO (left) and $\mathcal{A}_{\rm CP}(B^\pm \to \rho^\pm\gamma)$ (right) with $\epsilon_A = -0.3$ in the SM (solid line), and in the MSSM with $(\tan\beta, \mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}) = (3, 0.95)$ (dot-dashed line), $(\tan\beta, \mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}) = (30, 0.8)$ (dashed line) and $(\tan\beta, \mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff({\rm SM})}) = (30, -1.2)$ (dotted line). The SM and MSSM curves correspond respectively to $|V_{ub}/V_{td}| = 0.48$ and $|V_{ub}/V_{td}| = 0.63$, which are the central values in the CKM fits [15]. The allowed ranges for α in the MSSM from these fits for $f = 0.6$ are also indicated.

Once again, this would be a clear signal of supersymmetry in the large $\tan\beta$ domain.

In summary, we have examined the effects of supersymmetry on two observables of $B \to \rho\gamma$ decays: the isospin-violating quantity Δ and the direct CP asymmetry $\mathcal{A}_{\rm CP}$. We find that, in MSSM models, the predictions for these quantities can be substantially modified. In particular, over a large region of parameter space, the signs of Δ and $\mathcal{A}_{\rm CP}$ may be flipped, which would be a clear signal of new physics, and would point directly to the presence of supersymmetry.

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